Two types of conservation laws. Connection of physical fields with material systems. Peculiarities of field theories

L. I. Petrova

Abstract

Historically it happen so that in branches of physics connected with field theory and of physics of material systems (continuous media) the concept of "conservation laws" has a different meaning. In field theory "conservation laws" are those that claim the existence of conservative physical quantities or objects. These are conservation laws for physical fields. In contrast to that in physics (and mechanics) of material systems the concept of "conservation laws" relates to conservation laws for energy, linear momentum, angular momentum, and mass that establish the balance between the change of physical quantities and external action.

In the paper presented it is proved that there exist a connection between of conservation laws for physical fields and those for material systems. This points to the fact that physical fields are connected with material systems.

Such results has an unique significance for field theories. This enables one to substantiate many basic principles of field theories, such as, for example, the unity of existing field theories and the causality. The specific feature of field theory equations, namely, their connection to the equations for material systems, is elicited.

Such results have been obtained by using skew-symmetric differential forms, which reflect the properties of conservation laws.

(It appears to be possible to obtain radically new results due to using skew-symmetric differential form, which, in contrast to exterior forms, are defined on nonintegrable manifolds and possesses evolutionary properties. The mathematical apparatus of evolutionary forms includes nontraditional elements, that are, nonidentical relations and degenerate transformations. This enables one to describe the mechanism of arising and generating the structures that is impossible to perform within the framework of any existing mathematical formalisms. The existence of such skew-symmetric differential forms has been established by the author while studying the stability problems.)

1 Meaning of the concept of "conservation laws"

Due to the development of science the concept of "conservation laws" in thermodynamics, physics and mechanics contains different meanings.

In branches of physics related to field theory and in theoretical mechanics "the conservation laws" are those according to which there exist conservative physical quantities or objects. These are the conservation laws that were called above as "exact" ones.

In mechanics and physics of continuous media the concept of "conservation laws" relates to conservation laws for energy, linear momentum, angular momentum,

mentum, and mass, which establish the balance between the change of physical quantities and external action. These are balance conservation laws.

In thermodynamics conservation laws are associated with the principles of thermodynamics.

Thus, the concept of "conservation laws" is connected with exact conservation laws, i.e. balance conservation laws and some regularities expressed using the principles of thermodynamics.

Below it will be shown that the balance conservation laws are those for continuous media (material systems). The exact conservation laws are conservation laws for physical fields.

In addition it will be shown that the balance and exact conservation laws are related to each other. (The principles of thermodynamics integrate two balance conservation laws, namely, the balance conservation law for energy and that for linear momentum.)

The connection between conservation laws for physical fields and those for material systems discloses the peculiarity of physical fields, namely, their connection to material systems.

This has an unique importance for field theory as well. It will be shown that the equations of field theory, which are based on the conservation laws for physical fields, are connected with the equations that describe conservation laws for material systems. This provides answers to many problems of existing field theories.

The mathematical apparatus of skew-symmetric differential forms allows to describe conservations laws and their peculiarities.

2 Closed exterior skew-symmetric differential forms: Conservation laws for physical fields. Physical structures

2.1 Conservation laws for physical fields. Physical structures

As it was already pointed out, the conservation laws for physical fields are those that state the existence of conservative physical quantities or objects. [The physical fields [1] are a special form of substance, they are carriers of various interactions such as electromagnetic, gravitational, wave, nuclear and other types of interactions.]

The conservation laws for physical fields are described by closed exterior differential forms [2,3].

From the closure condition of closed exterior differential forms $d\theta^p$

$$d\theta^p = 0 \tag{1}$$

one can see that the closed form is a conservative quantity. This means that this form can correspond to conservation law, namely, to some conservative physical quantity.

If the form is closed on a pseudostructure only (closed inexact form), the closure conditions is written as

$$d_\pi \theta^p = 0 \tag{2}$$

$$d_\pi {}^*\theta^p = 0 \tag{3}$$

where ${}^*\theta^p$ is the dual form.

From conditions (2) and (3) one can see that the form closed on pseudostructure is a conservative object, namely, this quantity conserves on pseudostructure. This can also correspond to some conservation law, i.e. to conservative object.

The closed (inexact) exterior form and dual form describe the differential-geometrical structure. It is evident that such a structure (a conservative object) must correspond to exact conservation law. Such structures (pseudostructures with a conservative physical quantity), which correspond to exact conservation law, describe the physical structures that made up physical fields.

The problem of how physical structures arise and how physical fields are formatted will be discussed below.

The equations for physical structures ($d_\pi\, \theta^p = 0$, $d_\pi\, {}^*\theta^p = 0$) turn out to coincide with the mathematical expression for exact conservation law.

The mathematical expression for exact conservation law and its relation to physical fields can be schematically written in the following manner

$$\left\{\begin{array}{l} d_\pi \theta^p = 0 \\ d_\pi {}^*\theta^p = 0 \end{array}\right. \quad \mapsto \quad \left\{\begin{array}{l} \theta^p \\ {}^*\theta^p \end{array}\right. \quad \text{— physical structures} \quad \mapsto \quad \text{physical fields}$$

It is seen that the exact conservation law is that for physical fields.

(It should be emphasized that the closed *inexact* forms correspond to the physical structures that made up physical fields. The *exact* forms correspond to the material system *elements*.)

2.2 Closed inexact exterior forms as the basis of field theories

The field theories, i.e. the theories that describe physical fields, are based on the properties of closed inexact exterior differential and dual forms that correspond to exact conservation laws.

The invariant properties of closed exterior differential forms reveal themselves explicitly or implicitly in practically all formalisms of field theories such as the Hamilton formalism, tensor approaches, group methods, quantum mechanic equations, the Yang-Mills theory and so on.

The nondegenerate transformations of field theories are those of closed exterior forms.

Since the closed form is a differential (a total one if the form is exact, or an interior one on the pseudostructure if the form is inexact), it is obvious that the closed form turns out to be invariant under all transformations that conserve the differential. The unitary, tangent, canonical and gradient transformations and

so on are examples of such transformations of closed exterior forms. *These are gauge transformations of field theories.* The gauge transformations for spinor, scalar, vector and tensor fields are respectively transformations of closed (0-form), (1-form), (2-form) and (3-form).

The gauge, i.e. interior, symmetries of field theories (which correspond to gauge transformations) are symmetries of closed exterior forms. Exterior symmetries of the field theory equations are those of closed dual forms.

Operators of the field theory are connected to nondegenerate transformations of closed exterior forms. In terms of operators d (exterior differential), δ (the operator of transformation that converts the form of degree $p+1$ into the form of degree p), δ' (for cotangent transformations), Δ (for the transformation $d\delta-\delta d$), δ (for transformation $d\delta' - \delta' d$) one can write the operators of field theory equations. In terms of these operators, which act to exterior forms, it is possible to write the Green, d'Alambert, Laplace operators, as well as the operator of canonical transformation [4].

One can make sure that all existing field theories were built on the postulates of invariance and covariance, which are closure conditions for exterior and dual forms.

And there is the following correspondence.

-Closed exterior forms of zero degree correspond to quantum mechanics.

-The Hamilton formalism bases on the properties of closed exterior and dual forms of first degree.

-The properties of closed exterior and dual forms of second degree are at the basis of the equations of electromagnetic field.

-The closure conditions of exterior and dual forms of third degree form the basis of equations for gravitational field.

Closed inexact exterior or dual forms are solutions of the field theory equations.

One can see that field theory equations as well as nondegenerate transformations of field theories are connected with closed exterior forms of a certain degree. This enables one to introduce the classification of physical fields and interactions in degrees of closed exterior forms. (If to denote the degree of closed exterior forms by k, then $k = 0$ corresponds to strong interaction, $k = 1$ does to weak one, $k = 2$ does to electromagnetic one, and $k = 3$ corresponds to gravitational interaction.)

Such a classification shows that there exists an internal connection between field theories, which describe physical fields of various types. It is evident that the degree of closed exterior forms is a parameter that integrates field theories into unified field theory.

A significance of exterior differential forms for field theories consists in the fact that they disclose the properties that are common for all field theories and physical fields irrespective of their specific type. This is a step to building a unified field theory.

However, the theory of exterior forms cannot solve the problem of justifying the principles that are at the basis of field theories. To solve these problems, one has to answer the following questions:

(a) how are obtained the closed and dual forms that made up physical structures and on the properties of which the field theories are based;

(b) what is a physical meaning of such a parameter like a degree of closed exterior forms, which unite field theories;

(c) by what are conditioned the symmetries of closed exterior and dual forms that are assigned to internal and external symmetries of field theories;

(d) with what are connected the transformations of closed exterior forms, that is, with what are connected the gauge transformations in field theories;

(e) how to explain a discrete realization of closed (inexact) exterior forms that could disclose the quantum nature of field theories.

Within only framework of exterior differential forms it is impossible to answer these questions. Below it will be shown that closed exterior forms, whose properties lie at the basis of field theories, are realized from evolutionary forms obtained from the equations that describe conservation laws for material systems. This enables one to understand the basic principles of field theories, namely, their connections with the equations for material systems, and answer the questions posed up.

3 Evolutionary differential forms: Conservation laws for material systems. Noncommutativity of conservation laws for material systems.

The properties of balance conservation laws, as well as the properties of exact conservation laws, are described by skew-symmetric differential forms. But these skew-symmetric forms, as contrasted to exterior forms, are defined on nonintegrable manifolds and possess the evolutionary properties [5]. [As examples of manifolds, with which the evolutionary forms are connected, are the tangent manifolds of differential equations that describe any processes, the Lagrangian manifolds, the manifolds made up by trajectories of material system elements (particles), which are obtained while describing the evolutionary processes in material media, and others.]

(The evolutionary forms have an unique peculiarity. From the evolutionry form obtained from the equations describing the balance conservation laws for material systems (continuous media) the closed exterior forms, which describe the conservation laws for physical fields, are realized. This means that physical fields are connected with material systems.)

3.1 Conservation laws for material systems

The conservation laws for material systems (material media) are conservation laws of energy, linear momentum, angular momentum, and mass. In contrast to conservation laws for physical fields these conservation laws are balance ones.

[The material system is a variety of elements which have internal structure, move and interact to one another. Examples of elements that constitute material system are electrons, protons, neutrons, atoms, fluid particles, cosmic objects and others. As examples of material

systems it may be thermodynamic, gas dynamical, cosmic systems, systems of elementary particles (pointed above) and others. The physical vacuum in its properties may be regarded as an analogue of material system that generates some physical fields. Any material media are such material systems]

In mechanics and physics of material systems (of continuous media) the equations of balance conservation laws are used for description of physical quantities, which specify the behavior of material systems. However, it turns out that the role of these equations is much wider. They describe evolutionary processes in material systems that are accompanied by an origin of physical structures, from which physical fields are formatted.

The equations of balance conservation laws are differential or integral equations [6-8]. (The equations of mechanics and physics of continuous media such as the Euler and Navier-Stokes equations [7] are examples.)

The functions sought being solutions to equations of material media are usually functions which relate to such physical quantities like a particle velocity (of elements), temperature or energy, pressure and density. Since these functions relate to one material system, it has to exist a connection between them. This connection is described by some state functional (from which the state function is obtained). From the equations of balance conservation laws one gets the relation for state functional that, as it will be shown below, is of great significance both in mathematical physics, which describes material systems, and in field theories, which describe physical fields.

3.2 Analysis of the equations of balance conservation laws. Evolutionary relation.

It appears that, even without a knowledge of concrete form of the balance conservation law equations, with the help of skew-symmetric differential forms one can see specific features of these equations that elucidate the properties of balance conservation laws and their role in evolutionary processes.

The functional properties of equations or sets of equations depend on whether or not the derivatives of differential equations or of the equation in the set of differential equations are conjugated. [The necessity of studying the conjugacy of equations describing any process has a physical meaning. If these equations (or derivatives with respect to different variables) be not conjugated, the solutions to corresponding equations prove to be noninvariant, that is, they are functionals rather then functions. The realization of the conditions (while varying variables), under which the equations become conjugated ones, leads to that the relevant solution becomes invariant. It will be shown below that the transition to invariant solution describes the mechanism of evolutionary transition from one quality to another, which leads to emergence of differential-geometrical structures].

Equations are conjugate if they can be contracted into identical relations for differential, i.e. for a closed form.

Let us analyze the equations that describe the balance conservation laws for energy and linear momentum.

We introduce two frames of reference: the first is an inertial one (this frame of reference is not connected with material system), and the second is an accom-

panying one (this system is connected with the manifold built by trajectories of material system elements). The energy equation in the inertial frame of reference can be reduced to the form:

$$\frac{D\psi}{Dt} = A \tag{4}$$

where D/Dt is the total derivative with respect to time, ψ is the functional of the state that specifies material system, A is the quantity that depends on specific features of the material system and on external energy actions onto the system. {The action functional, entropy, wave function can be regarded as examples of the functional ψ. Thus, the equation for energy expressed in terms of the action functional S has a similar form: $DS/Dt = L$, where $\psi = S$, $A = L$ is the Lagrange function. In mechanics of continuous media the equation for energy of ideal gas can be expressed in the form [6]: $Ds/Dt = 0$, where s is entropy. In this case $\psi = s$, $A = 0$. It is worth noting that the examples presented demonstrate that the action functional and entropy play the same role.}

In accompanying frame of reference the total derivative with respect to time is converted into a derivative along trajectory. Equation (4) is now written in the form

$$\frac{\partial\psi}{\partial\xi^1} = A_1 \tag{5}$$

here ξ^1 is the coordinate along trajectory ($A_1 = A$).

In a similar manner, in accompanying frame of reference the equation for linear momentum appears to be reduced to the equation of the form

$$\frac{\partial\psi}{\partial\xi^\nu} = A_\nu, \quad \nu = 2, \ldots \tag{6}$$

where ξ^ν are the coordinates in the direction normal to trajectory, A_ν are the quantities that depend on specific features of the system and external force actions.

Eqs. (5), (6) can be convoluted into the relation

$$d\psi = A_\mu \, d\xi^\mu, \quad (\mu = 1, \nu) \tag{7}$$

where $d\psi$ is the differential expression $d\psi = (\partial\psi/\partial\xi^\mu)d\xi^\mu$.

Relation (7) can be written as

$$d\psi = \omega \tag{8}$$

here $\omega = A_\mu \, d\xi^\mu$ is the differential form of first degree.

Since the balance conservation laws are evolutionary ones, the relation obtained is also an evolutionary relation.

Relation (8) was obtained from the equations of balance conservation laws for energy and linear momentum. In this relation the form ω is that of first degree. If the equations of balance conservation laws for angular momentum be added to the equations for energy and linear momentum, this form in evolutionary

relation will be a form of second degree. And in combination with the equation of balance conservation law of mass this form will be a form of degree 3.

Thus, in general case the evolutionary relation can be written as

$$d\psi = \omega^p \tag{9}$$

where the form degree p takes the values $p = 0, 1, 2, 3$. (The evolutionary relation for $p = 0$ is similar to that in differential forms, and it was obtained from interaction of energy and time.)

In relation (8) the form ψ is a form of zero degree. And in relation (9) the form ψ is a form of $(p-1)$ degree.

3.3 Nonidentity of evolutionary relation: Noncommutativity of balance conservation laws.

Let us show that *the evolutionary relation obtained from the equation of balance conservation laws proves to be nonidentical one.*

To do so we shall analyze relation (8).

The relation may be identical one if this is the relation between measurable (invariant) quantities or between observable (metric) objects, in other words, between quantities or objects that are comparable.

In the left-hand side of evolutionary relation (8) there is a differential that is a closed form. This form is an invariant object. The right-hand side of relation (8) involves the differential form ω, that is not an invariant object because in real processes, as it is shown below, this form proves to be unclosed.

For the form be closed the differential of the form or its commutator must be equal to zero (the elements of the form differential are equal to components of its commutator).

Let us consider the commutator of the form $\omega = A_\mu d\xi^\mu$. The components of commutator of such a form can be written as follows:

$$K_{\alpha\beta} = \left(\frac{\partial A_\beta}{\partial \xi^\alpha} - \frac{\partial A_\alpha}{\partial \xi^\beta}\right)$$

(here the term connected with a nonintegrability of the manifold has not yet been taken into account).

The coefficients A_μ of the form ω have been obtained either from the equation of balance conservation law for energy or from that for linear momentum. This means that in the first case the coefficients depend on energetic action and in the second case they depend on force action. In actual processes energetic and force actions have different nature and appear to be inconsistent. The commutator of the form ω made up of derivatives of such coefficients is nonzero. This means that the differential of the form ω is nonzero as well. Thus, the form ω proves to be unclosed and is not a measurable quantity.

This means that the evolutionary relation involves an unmeasurable term. Such a relation cannot be identical one. (In the left-hand side of this relation it stands a differential, whereas in the right-hand side it stands an unclosed

form that is not a differential.) [Nonidentical relation was analyzed in paper J.L.Synge "Tensorial Methods in Dynamics" (1936). And yet it was allowed a possibility to use the sign of equality in nonidentical relation.]

[The nonidentity of evolutionary relation does not mean that the mathematical description of physical processes is not perfectly exact. The nonidentity of the relation means that the derivatives, whose values correspond to real values in physical processes, cannot be consistent (their mixed derivatives cannot be commutative; the commutator is nonzero) because they are obtained at the expense of external action and are unmeasurable quantities.]

The nonidentity of evolutionary relation means that the equations of balance conservation laws turn out to be nonconjugated (and hence, nonintegrable: one cannot convolute them into an identical relation and obtain a differential). And this indicates that the balance conservation laws are noncommutative.

As noted above, each balance conservation law depends on relevant action (that the material system is subjected to). So, the conservation law for energy depends on energetic action, the conservation law for linear momentum depends on force action, and so on. In actual processes the energetic and the force actions have different nature, and this is the cause of noncommutativity of balance conservation laws. The balance conservation laws that depend on actions of different nature cannot be commutative.

Thus, the nonidentity of evolutionary relation (see, formulas (8), (9)) means that the balance conservation law equations are inconsistent. And this indicates that the balance conservation laws are noncommutative. (If the balance conservation laws be commutative, the equations would be consistent and the evolutionary relation would be identical).

{The nonidentity of evolutionary relation are connected with the differential form ω^p that enters into this relation. This is a skew-symmetric form with the basis, in contrast to the basis of exterior form, that is a deforming (nonintegrable) manifold. The peculiarity of skew-symmetric forms defined on such manifold is the fact that their differential depends on the basis. The commutator of such form includes the term that is connected with differentiating the basis. This can be demonstrated by an example of the first-degree skew-symmetric form.

Let us consider the first-degree form $\omega = a_\alpha dx^\alpha$. The differential of this form can be written as $d\omega = K_{\alpha\beta}dx^\alpha dx^\beta$, where $K_{\alpha\beta} = a_{\beta;\alpha} - a_{\alpha;\beta}$ are the components of commutator of the form ω, and $a_{\beta;\alpha}$, $a_{\alpha;\beta}$ are the covariant derivatives. If we express the covariant derivatives in terms of connectedness (if it is possible), they can be written as $a_{\beta;\alpha} = \partial a_\beta/\partial x^\alpha + \Gamma^\sigma_{\beta\alpha}a_\sigma$, where the first term results from differentiating the form coefficients, and the second term results from differentiating the basis. If we substitute the expressions for covariant derivatives into the formula for commutator components, we obtain the following expression for the commutator components of the form ω:

$$K_{\alpha\beta} = \left(\frac{\partial a_\beta}{\partial x^\alpha} - \frac{\partial a_\alpha}{\partial x^\beta}\right) + (\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta})a_\sigma$$

Here the expressions $(\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta})$ entered into the second term are just components of commutator of the first-degree metric form that specifies the manifold deformation and hence is nonzero. (In commutator of the exterior form, which is defined on differentiable manifold the second term is not present.) [It is well-known that the metric form commutators of the first-, second- and third degrees specifies, respectively, torsion, rotation and curvature.

The skew-symmetric differential forms defined on nonintegrable manifolds are evolutionary forms. They are obtained while describing some processes. }

In the next section by the analysis of nonidentical evolutionary relation it will be shown that the noncommutativity of balance conservation laws is a

driving force of evolutionary processes that proceed in material systems and are accompanied by generation of physical structures.

4 Physical significance of nonidentical evolutionary relation: Relation of conservation laws for physical fields to conservation laws for material systems. Relation of physical fields to material systems

4.1 Nonidentity of evolutionary relation: nonequilibrium state of material system

The noncommutativity of balance conservation laws is a characteristics of the state of material system. This is reflected by the evolutionary relation.

In the left-hand side of evolutionary relation (see, (8, 9)) there is the functional expression $d\psi$ that determines the state of material system.

It is evident that if the balance conservation laws be commutative, the evolutionary relation would be identical and from that it would be possible to get the differential $d\psi$ and find the state function ψ, this would indicate that material system is in equilibrium state. However, as it has been shown, in real processes the balance conservation laws are noncommutative. The evolutionary relation is not identical and from this relation one cannot get the differential $d\psi$. The absence of differential $d\psi$ points to the fact that the material system state is nonequilibrium. (As it will be shown below, the interior (on pseudostructure) differential $d\psi$ can be realized and this will correspond to locally equilibrium state.)

The evolutionary relation possesses one more peculiarity, namely, this relation is a selfvarying relation. (The evolutionary form entering into this relation is defined on deforming manifold made up by trajectories of material system elements. This means that the evolutionary form basis varies. In turn, this leads to variation of evolutionary form, and the process of intervariation of evolutionary form and of the basis is repeated.)

Selfvariation of nonidentical evolutionary relation points to the fact that the nonequilibrium state of material system turns out to be selfvarying. State of material system changes but remains nonequilibrium during this process.

It is evident that selfvariation of nonequilibrium state of material system proceeds under the action of internal force whose quantity is described by the commutator of unclosed evolutionary form ω^p. (If the evolutionary form commutator be zero, the evolutionary relation would be identical, and this would point to the equilibrium state, i.e. the absence of internal forces.) Everything that gives a contribution into the evolutionary form commutator leads to emergence of internal force.

Is the transition from nonequilibrium state to equilibrium one possible?

Since the evolutionary form is unclosed, the evolutionary relation cannot be identical. This means that the nonequilibrium state of material system holds. However, from the evolutionary relation it can be obtained identical relation on some structure (more exactly, on pseudostructure). And this will correspond to transition of material system to locally equilibrium state.

To obtain an identical relation from evolutionary nonidentical relation, it is necessary that a closed exterior differential form should be derived from evolutionary differential form that is included into nonidentical relation. A transition from evolutionary form (whose differential is *nonzero*) to closed exterior forms (whose differential is *zero*) is possible only as degenerate transformation, that is, a transformation that does not conserve the differential.

Such a transformation is possible under realization of appropriate conditions.

The conditions of degenerate transformation are connected with symmetries caused by degrees of freedom of material system. The translational degrees of freedom, internal degrees of freedom of the system elements, and so on can be examples of such degrees of freedom. [To the degenerate transformation it must correspond a vanishing of some functional expressions, such as Jacobians, determinants, the Poisson brackets, residues and others. Vanishing of these functional expressions is a closure condition for dual form. And it should be emphasize once more that the degenerate transformation is realized as a transition from accompanying noninertial frame of reference to locally inertial system. The evolutionary form and nonidentical evolutionary relation are defined in noninertial frame of reference (deforming manifold). But the closed exterior form and the identical relation are obtained with respect to the locally-inertial frame of reference (pseudostructure). Mathematically this is described as a transition from noninertial frame of reference to inertial one.]

The conditions of degenerate transformation can be realized under selfvariation of evolutionary relation.

The realization of the conditions of degenerate transformation leads to realization of pseudostructure π (the closed dual form) and formatting the closed inexact form ω_π, whose closure conditions have the form

$$d_\pi \omega^p = 0, d_\pi{}^* \omega^p = 0 \tag{10}$$

[Pseudostructures specify the integral surfaces: the characteristics (the determinant of coefficients at the normal derivatives vanishes), the singular points (Jacobian is equal to zero), the envelopes of characteristics of Eulers' equations and so on.] {{Cohomology (de Rham cohomology, singular cohomology [9]), sections of cotangent bundles, integral and potential surfaces and so on may be regarded as examples of pseudostructures. The eikonal surfaces correspond to pseudostructures}

On the pseudostructure π from the evolutionary form ω^p it arises (under degenerate transformation) the closed inexact exterior form ω_π^p and from evolutionary relation (9) it is obtained the relation

$$d_\pi \psi = \omega_\pi^p \tag{11}$$

which proves to be an identical relation since the closed inexact form ω_π^p is a differential (interior on pseudostructure).

From identical relation one can obtain the state differential $d_\pi\psi$ and find the state function, and this points to that the material system state is a local equilibrium state.

Thus, from the properties of nonidentical evolutionary relation and those of evolutionary form one can see that under realization of the additional condition (which is a condition of degenerate transformation) the transition of material system state from nonequilibrium to locally equilibrium state can be realized.

But in this case the total state of material system turns out to be nonequilibrium because the evolutionary relation itself remains to be nonidentical one. The equilibrium state is realized only locally since the state differential is interior one defined exclusively on pseudostructure.

[Here one can see the simultaneous presence of identical and nonidentical relation. This peculiarity discloses the duality of the quantity ψ. From identical relation one can find the differential $d\psi$ on pseudostructure and after integrating obtain the value ψ. In this case ψ on pseudostructure becomes the state function. However, since the evolutionary relation remains to be nonidentical, this means that the total differential $d\psi$ is not defined and ψ occurs to be a certain functional. Such a duality one can trace by the example of entropy. Entropy can be functional, because it is impossible to find the entropy differential (total) from nonidentical relation, and be simultaneously the state function on pseudostructure if the interior (on pseudostructure) differential of entropy is realized.]

Under realization of new additional conditions a new identical relation can be obtained. As a result, the nonidentical evolutionary relation can generate identical relations.

4.2 Realization of physical structures that form physical fields. Connection of physical structures with material systems.

The emergence of the closed (on pseudostructure) inexact exterior form ω_π^p points to origination of physical structure. The closure conditions (10) for exterior inexact form correspond to conservation law, i.e. to existence of conservative on pseudostructure quantity, and describe a differential-geometrical structure. These are such structures (pseudostructures with conservative quantities) that are physical structures, from which physical fields are formatted.

The transition from nonidentical relation (9) obtained from balance conservation laws equations to identical relation (11) means the following. Firstly, an emergence of closed (on pseudostructure) inexact exterior form (right-hand side of relation (11)) points to origination of physical structure. And, secondly, the existence of state differential (left-hand side of relation (11) points to transition of material system from nonequilibrium state to locally-equilibrium state.

One can see that the transition of material system into locally equilibrium state is accompanied by origination of physical structures. Massless particles, charges, structures made up by eikonal and potential surfaces, wave fronts, and so on are examples of physical structures.

(Here it should pay attention to the fact that to every physical field it is

assigned its own material system. At present the question of what material system is assigned to given physical field remains unsolved. As it was mentioned before, the thermodynamical, gas dynamical, cosmological systems, the systems of charged particles and others are examples of material systems. Maybe, the physical vacuum is such material system for elementary particles.)

Identical relation (11) holds the duality. The left-hand side of this relation includes the differential, which specifies material system and whose availability points to locally-equilibrium state of material system. And the right-hand side includes the closed inexact form, which is a characteristics of physical structures and from which the physical fields are formatted.

Such a duality of identical relation (11) emphasizes its unique nature. This relation demonstrates the connection between material systems and physical fields. And this connection is given up by the evolutionary process, which is described by nonidentical evolutionary relation obtained from the equations of balance conservation laws for material systems.

The duality of identical relation also explains the duality of nonidentical evolutionary relation. On the one hand, evolutionary relation describes the evolutionary process in material systems, and on the other describes the process of generating physical fields.

The origination of physical structures in evolutionary process is connected with discrete changes of measurable (inherent) quantities of material system and reveals in material system as an emergence of certain observable formations, which develop spontaneously. Such formations and their manifestations are fluctuations, turbulent pulsations, waves, vortices, and others. The spontaneous emergence of observable formations in material system explains the mechanism of such processes like the development of instability, the advent of vorticity [10], the origination of turbulence.

The observed formation and the physical structure are not identical objects. If the wave be such a formation, the element of wave front made up the physical structure at its motion. The structures of physical fields and the formations of material systems observed are a manifestation of the same phenomena. The light is an example of such a duality. The light manifests itself in the form of massless particle (photon) and of a wave.

(This duality also explains a distinction in studying the same phenomena in material systems and physical fields. In the physics of continuous media (material systems) the interest is expressed in generalized solutions to the equations of balance conservation laws. These are solutions that describe the formations in material media observed. The investigation of relevant physical structures is carried out using the field theory equations.)

Thus, it has been shown that there exists a connection of physical fields with material systems. Material systems generate physical structures that made up physical fields. In this case the process of generation is controlled by conservation laws for material systems. Such controlling role of balance conservation laws in evolutionary processes is explained by noncommutativity of conservation laws.

[*The noncommutativity of balance conservation laws and their governing role in evolutionary processes, that are accompanied by emerging physical structures, practically have not been taken into account in the explicit form anywhere. The mathematical apparatus of evolutionary differential forms enables one to take into account and describe these points.*]

Since physical fields are connected with material systems, it seems natural to assume it has to be a connection between field theories (those that describe physical fields) and the equations for material systems (the equations of balance conservation laws for energy, linear momentum, angular momentum, and mass as well as the analog of such laws for time, which takes into account the noncommutativity of time and energy of material systems). In particular, it has to exist a connection of the field theory equations with the equations for material systems. And this connection has to be described by closed exterior and evolutionary skew-symmetric forms, which correspond to conservation laws for physical fields and material systems.

5 Peculiarities of field theories

5.1 Substantiation of basic principles of existing field theories

In subsection 2.2 it has been shown that the properties of closed exterior forms, which correspond to conservation laws for physical fields, lie at the basis of existing field theories. The results obtained from the analysis of the equations of conservation laws *for material systems* allows to substantiate these positions.

It should be underlined the following results, which give the answer to the questions posed in subsection 2.2.

a). Closed exterior forms on which properties the theories describing physical fields are based, are obtained from evolutionary forms entering into nonidentical relation derived from the equations of balance conservation laws for material systems.

b). As it was shown above the degree of closed exterior forms, which plays a role of classification parameter for physical fields, is a degree of closed inexact exterior forms realized from evolutionary form in nonidentical relation following from the equations of noncommutative balance conservation laws for material systems. From evolutionary forms, whose degree p relates to the number of interacting balance conservation laws and can take the values $0, 1, 2, 3$, the closed (inexact) forms of degrees k realize and can take the values $p, p-1, ..., 0$.

From this one can see that the degree of closed exterior forms, which plays a role of parameter for united field theory. is connected with the number of the equations of interacting noncommutative balance conservation laws and can change from $k = 0$ to $k = 3$.

c). The external symmetries of the equations of field theory are symmetries of closed dual forms. Such symmetries are conditioned by degrees of freedom of material system (translational, rotational, oscillatory and so on). Hence, the

external symmetries of the equations of field theory are also conditioned by degrees of freedom of material system.

The gauge, i.e. internal, symmetries of field theory (corresponding to gauge transformations) are those of closed exterior forms. The symmetries of closed exterior forms are obtained from the evolutionary form coefficients and therefore is connected with the characteristics of material system.

As the result, the symmetries of closed exterior forms and, consequently, the interior symmetries of field theory are defined by the characteristics of material system.

d). The gauge transformations in field theories, which are nondegenerate transformations of closed exterior forms, are connected with degenerate transformations of evolutionary forms obtained from the equations of conservation laws for material systems. (Nondegenerate transformations of closed exterior forms is a transition on integrable manifold from any closed inexact form to another closed inexact form. However, every of these closed inexact forms is obtained from evolutionary form defined on nonintegrable manifold by using degenerate transformation.)

e). Since physical fields, as it has been shown, are formed up by physical structures, this means that physical fields are discrete ones rather then continuous. The discreteness of physical fields points to the fact that field theories mast be quantum ones. This explains the quantum character of field theories.

Characteristics of physical structures are connected with the characteristics of material systems. From this it follows that the constants of field theory must be connected with parameters of material systems.

The results that prove the basic principles of field theories were obtained while studying the equations of conservation laws for material systems. This points to the fact that the equations of field theory must be connected with the equations for material systems. Such a connection, which will be described in next subsection, clarify the specific features of the field theory equations.

5.2 Connection between the equations of field theory and the equations for material systems

Before discussion of the equations of existing field theories it should call attention to functional peculiarities of the equations of mathematical physics and to nonidentical evolutionary relation.

Specific features of the equations of mathematical physics

We will point out some peculiarities of the equations for material systems and those for field theories.

The equations for material systems are equations of balance conservation laws of energy, linear momentum, angular momentum, and mass. These are the equations of mechanics of continuous media, as well as the equations of continuous medium physics, which describe physical processes. In physics the

interest is in generalized solutions (which depend only on variables) that describe observed quantities (rather then the process itself). To do this, it is necessary to find the condition of integrability of the system of differential equations. That is, it is necessary to investigate the functional relation (nonidentical evolutionary relation) obtained from differential equations.

The goal of continuous medium mechanics is to describe the process of variation of continuous medium. In this case for solving differential equations the numerical methods are commonly used without analyzing the conditions of integrability of these equations. This leads to that the solutions to equations depend not only on variables but also on the path of integration.

As the result one has that in physics only generalized solutions (which are functions only of variables) are considered and isn't considered the solutions to original equations, which are not generalized solutions, and in mechanics the generalized solutions (for obtaining which one must investigate the integrability of equations) are not separated out. Such limited approach both in physics and in mechanics lead to nonclosure of relevant theories. Without finding generalized solutions in mechanics it is impossible to describe such processes like turbulence, emergence of vorticity and so on (i.e. the processes of emerging any formations that can be described by only generalized solutions). And without accounting for solutions that are not generalized ones in physics it is impossible to describe the reason of obtaining generalize solutions.

Whereas the distinction between mechanics and physics of continuous medium consists in that both in mechanics and physics different types of solutions to the same system of equations are used, the distinction between physics of continuous medium and physics, which describes physical fields (i.e. field theory), is connected with use of distinct differential equations.

The equations for material systems and the field theory equations are radically distinguished types of differential equations.

The fundamental difference between these two types is due to the fact that the solutions to differential equations for material systems have to describe physical *quantities* (of material systems), whereas the solutions to field theory equations have to describe physical *structures* (formatting physical fields). The basic element of the equations for material systems are *derivatives*, by integration of which one can obtain the desired functions describing physical quantities. In contrast to that, the properties of differential forms, that are, *differential expressions and differentials* are used as the basis of field theory equations. This is explained by the fact that the physical structures, from which physical fields are made up, are described by closed exterior forms, that is, by differentials.

Thus, it should to distinguish two types of equations of mathematical physics: the differential equations for material systems and the equations of field theory. (This explains a distinction in studying the same phenomena in material systems and physical fields. In the physics of continuous medium (material systems) the solutions to equations (which are generalized solutions to equations of balance conservation law) describe the observed formations in material media. The investigation of relevant physical structures is carried out using the solutions to field theory equations.)

It appears that these two types of equations are mutually connected.

It was shown that closed exterior forms, which describe physical fields and, hence, must be the solutions to field theory equations, follow from the nonidentical evolutionary relation, which is derived from the equations for material systems. This means that there exists a connection between field theory equations and the equations for material systems. And this connection is realized with the help of nonidentical evolutionary relation.

Functional peculiarities of nonidentical evolutionary relation

As it is shown (see formula (9)) the evolutionary relation is derived from the equations of balance conservation laws of energy, linear momentum, angular momentum, and mass and in general case (as it was shown) can be written as

$$d\psi = \omega^p \tag{9}$$

where ψ are *functionals* like wave-function, action functional, entropy, the Pointing vector and others, ω^p is the evolutionary form of degree p, which depends on the characteristics of material system and the external action to the system, the form degree p takes the values $p = 0, 1, 2, 3$, which are connected with the number of interacting noncommutative balance conservation laws.

The nonidentical evolutionary relations are relations in differential forms for *functionals*.

Such functionals like wave-function, action functional, entropy, and others are used both in field theory and in the theories that describe material systems (in mechanics and physics of continuous medium). As it is seen from the analysis of nonidentical relation in section 4, in the theories that describe material systems these functionals specify the state of material system. And in field theory they describe physical fields – the field theory equations are those for such functionals. That is, such functionals possess a duality. This duality of functionals and relevant nonidentical relations just allows to disclose the connection between the field theory equations, which describe physical fields, and the equations for material systems.

The equations for material systems (the equations of balance conservation laws for energy, linear momentum, angular momentum, and mass) are partial differential equations for desired functions like the velocity of particles (elements), temperature, pressure and density that correspond to physical quantities of material systems (continuous media). The functionals like wave-function, action functional, entropy and others and corresponding nonidentical relations in continuous medium physics are used only for analysis of integrability of differential equations, that is, as a tool for obtaining the required solutions. [The analysis of integrability of differential equations is necessary for to obtain the generalized solution that describes measurable physical quantities of material system. The identity of relation obtained from evolutionary equation means that original equations for material system (the equations of conservation laws) become consistent and integrable on pseudostructures. Identity of the relation obtained from evolutionary relation means that the original equations

for material system become consistent and integrable on pseudostructures. Pseudostructures made up integral surfaces (such as characteristics, potential surfaces and others) on which the desired quantities of material system (such as temperature, pressure, density) become functions of only variables and don't depend on the path of integration. These are generalized solutions).]

The field theory equations are those that describe physical fields. Since physical fields are made up by physical structures, which are described by closed exterior *inexact* forms, it is obvious that solutions to the field theory equations must be closed exterior forms, i.e. to be differentials. And such differentials, which are closed exterior forms, can be obtained from nonidentical evolutionary relation for functionals. This means that in field theory the nonidentical evolutionary relations for functionals can play a role of field theory equation. (As it will be shown later, in fact all equations of existing field theories are the analog to such relation or its differential or tensor representation.)

It is seen that the nonidentical evolutionary relation possesses the duality: this relation is used both in mechanics and physics of continuous medium and in field theory. The nonidentical relation unifies existing theories, namely, the field theory and the theory of continuous medium, and make them closed ones.

Role of nonidentical evolutionary relation as the equation of general field theory

If the nonidentical evolutionary relation be regarded as the equation for deriving identical relation with including closed forms (describing physical structures desired), one can see that there is a correspondence between such evolutionary relation and the equations of existing field theories.

As it was already pointed out, at the basis of all existing field theories there lie the properties of closed exterior forms that correspond to conservation laws for physical fields. Since the conservation laws are described by closed inexact exterior and dual forms, it is evident that relevant closed exterior and dual forms must be found from the field theory equations. One can ensure that the identical relations, from which relevant closed exterior and dual forms follow, are the solutions to equations of all existing field theories.

The peculiarity of field theory equations consists in the fact that all these equations have the form of relations. They can be relations in differential forms or in the forms of their tensor or differential (i.e. expressed in terms of derivatives) analogs.

The Einstein equation is a relation in differential forms. This equation relates the differential of the first degree form (Einstein's tensor) and the differential form of second degree, namely, the energy-momentum tensor. (It should be noted that Einstein's equation is obtained from differential form of third degree).

The Dirac equation relates Dirac's *bra*- and *cket*- vectors, which made up the differential forms of zero degree.

The Maxwell equations have the form of tensor relations.

Field and Schrődinger's equations have the form of relations expressed in terms of derivatives and their analogs.

All equations of existing field theories have the form of nonidentical relations. Such peculiarity of the equations is explained by the fact that only equations that have the form of relations can have solutions being differentials rather then functions. From such equations it follows the identical relations, from which the closed forms, which describe physical structures, are found.

The identical relations, which include closed exterior forms or their tensor or differential analogs, are obtained from existing field theory equations.

By analyzing the field theory equations one can see that from the field theory equations it follows such identical relation like

1) the Dirac relations made up of Dirac's *bra*- and *cket*- vectors, which are connected with closed exterior form of zero degree [11];

2) the Poincare invariant $ds = -H\,dt + p_j\,dq_j$, which is connected with closed exterior form of first degree;

3) the relations $d\theta^2 = 0$, $d^*\theta^2 = 0$ for closed exterior forms of second degree obtained from Maxwell equations [4];

4) the Bianchi identities connected with skew-symmetric forms of third degree [12].

It turns out that all equations of existing field theories are in essence relations that connect skew-symmetric forms or their analogs. In this case one can see that the equations of field theories have the form of relations for functionals such as wave function (the relation corresponding to differential form of zero degree), action functional (the relation corresponding to differential form of first degree), the Pointing vector (the relation corresponding to differential form of second degree). The tensor functionals (such as the Riemann-Christoffel tensor, the Ricci tensor), that correspond to Einstein's equation, are obtained from the relation connecting the differential forms of third degree.

The nonidentical evolutionary relation derived from the equations for material system unites the relations for all these functionals. This is, all equations of field theories are an analog of nonidentical evolutionary relation. From this it follows that the nonidentical evolutionary relation can play a role of the equation of general field theory that discloses common properties and peculiarities of existing equations of field theory.

The correspondence between the equations of existing field theories and the nonidentical evolutionary relation has the mathematical and physical meaning. Firstly, this discloses the internal connection between all physical theories. And secondly, this shows what has to lie at the basis of general field theory, i.e. solves the problem of causality.

References

1. Encyclopedic dictionary of physical sciences. -Moscow, Sov. Encyc., 1984 (in Russian).

2. Cartan E., Les Systemes Differentials Exterieus ef Leurs Application Geometriques. -Paris, Hermann, 1945.

3. Schutz B. F., Geometrical Methods of Mathematical Physics. Cambrige University Press, Cambrige, 1982.

4. Wheeler J. A., Neutrino, Gravitation and Geometry. Bologna, 1960.

5. Petrova L. I., Role of exterior and evolutionary skew-symmetric differential forms in mathematical physics. http://arxiv.org/abs/math-ph/0510077, 2005.

6. Tolman R. C., Relativity, Thermodynamics, and Cosmology. Clarendon Press, Oxford, UK, 1969.

7. Clark J. F., Machesney M., The Dynamics of Real Gases. Butterworths, London, 1964.

8. Dafermos C. M. In "Nonlinear waves". Cornell University Press, Ithaca-London, 1974.

9. Bott R., Tu L. W., Differential Forms in Algebraic Topology. Springer, NY, 1982.

10. Petrova L. I., The mechanism of generation of physical structures. //Nonlinear Acoustics - Fundamentals and Applications (18th International Symposium on Nonlinear Acoustics, Stockholm, Sweden, 2008), New York, American Institute of Physics (AIP), 2008, pp.151-154.

11. Dirac P. A. M., The Principles of Quantum Mechanics. Clarendon Press, Oxford, UK, 1958.

12. Tonnelat M.-A., Les principles de la theorie electromagnetique et la relativite. Masson, Paris, 1959.